SU(6) and chiral SU(3) symmetry breaking

1973 J. Phys. A: Math. Nucl. Gen. 6 409

(http://iopscience.iop.org/0301-0015/6/3/014)

J. Phys. A: Math., Nucl. Gen., Vol. 6, March 1973. Printed in Great Britain. © 1973

SU(6) and chiral SU(3) symmetry breaking

R Delbourgo

Physics Department, Imperial College of Science and Technology, London SW7 2BZ, UK

MS received 24 October 1972

Abstract. SU(6) breaking is used to obtain the form of chiral SU(3) breaking. A very specific mixture of $(3,\bar{3})^{1+8}$, $(6,\bar{6})^{1}$ and $(8,8)^{1}$ chiral representations is predicted.

The nature of chiral SU(3) ⊗ SU(3) symmetry breaking has become a subject of debate. Since the original proposal (Gell-Mann *et al* 1968) that the Hamiltonian density H' primarily belongs to the $(3,\bar{3})\oplus(\bar{3},3)$ representation, it has been suggested that (8, 8) terms can provide significant corrections (Barnes and Isham 1970, Genz and Katz 1970, Sirlin and Weinstein 1972), and recently it has been proposed (Dittner *et al* 1972, Auvil 1972) that the $(6,\bar{6})\oplus(\bar{6},6)$ terms have no reason to be neglected either. In an effort to shed some light on the controversy we have attempted to use nonchiral SU(6)⊗SU(6) to provide some important clues as to the character of H', and we shall report on this here. In terms of the quark fields ψ which make up the supermultiplets we find that the predominant part of H' can be compactly expressed as

$$H' = c(\bar{\psi}\psi+\bar{\psi}^{3}\psi_{3})+c'\{(\bar{\psi}\gamma_{\mu}\gamma_{5}\psi)^{2}-(\bar{\psi}\gamma_{5}\psi)^{2}-\tfrac{1}{2}(\bar{\psi}\gamma_{5}\lambda^{i}\psi)^{2}\} \qquad (1)$$

where, for mesons $c = 0{\cdot}48$, $c' = 0{\cdot}12$ and for baryons $c = 0{\cdot}84$, $c' = 0{\cdot}46$ in GeV units. If we identify ψ with the quark field which generates the current algebra, then this leads to a very specific combination of $(3,\bar{3})^{1+8}$, $(6,\bar{6})^{1}$ and $(8,8)^{1}$ chiral terms. We give numerical details below and some of the simplest consequences.

Supermultiplet schemes adopt nonchiral SU(6) ⊗ SU(6) and classify the lowest lying mesons and baryons as $(6,\bar{6})$ and (56, 1) representations respectively. In quark field language we write

$$\Phi_{A}^{B}(p) = u_{A}(p)\bar{v}^{B}(p), \qquad \Psi_{(ABC)}(p) = u_{A}(p)u_{B}(p)u_{C}(p).$$

For particles at rest the labels $A = \alpha,a$ run through six values and one can pick out the usual particle wavefunctions from the (normalized) expansions

$$\Phi_{A}^{B} = \tfrac{1}{2}\delta_{\alpha}^{\beta}(\lambda^{0}\phi_{5}^{0}+\lambda^{i}\phi_{5}^{i})_{a}^{b}+\tfrac{1}{2}\boldsymbol{\sigma}_{\alpha}^{\beta}\cdot(\lambda^{0}\boldsymbol{\phi}^{0}+\lambda^{i}\boldsymbol{\phi}^{i})_{a}^{b}$$
$$\Psi_{(ABC)} = (2)^{-1/2}(\mathrm{i}\boldsymbol{\sigma}\sigma_{2})_{\alpha\beta}\cdot\boldsymbol{D}_{\gamma(abc)}+6^{-1}\{\epsilon_{abd}(\lambda^{i})_{c}^{d}(\mathrm{i}\sigma_{2})_{\alpha\beta}N_{\gamma}^{i}+\text{cyclic}\}.$$

We shall assume the existence of a Hamiltonian H_0 which leads to these SU(6) representations, such that the quarks which constitute the hadrons are effectively massless. The perturbation H' then includes the quark mass terms and then serves to generate all the masses of the composites in the theory.

Knowledge about SU(6) breaking has been available for some years (Harari and Rashid 1966, Fayyazzuddin and Rashid 1966, Akyeampong 1966). To a reasonable approximation the meson $(\text{mass})^2$ matrix is given by a sum of tensors†

$$\{1,1\}^1 \oplus [\{35^{(8,1)},1\}^8 + \{1,35^{(8,1)}\}^8] \oplus \{35^{(1,3)},35^{(1,3)}\}^1 \oplus \{35^{(8,1)},35^{(8,1)}\}^1 \oplus \{35^{(8,3)},35^{(8,3)}\}^1$$

and the baryon $(\text{mass})^2$ matrix is dominated by the tensors

$$\{1,1\}^1 \oplus [\{35^{(8,1)},1\}^8 + \{1,35^{(8,1)}\}^8] \oplus [\{405^{(8,1)},1\}^8 + \{1,405^{(8,1)}\}^8].$$

In terms of u and v spinors normalized to unity we can characterize the meson tensors by means of the linear combination‡

$$\mu_0^2(\bar{u}u+\bar{v}v)+\mu_1^2(\bar{u}^3u_3+\bar{v}^3v_3)+\mu_2^2(\bar{u}\boldsymbol{\sigma}u)\,.\,(\bar{v}\boldsymbol{\sigma}v)+\mu_3^2(\bar{u}\lambda^i u)(\bar{v}\lambda^i v)+\mu_4^2(\bar{u}\boldsymbol{\sigma}\lambda^i u)\,.\,(\bar{v}\boldsymbol{\sigma}\lambda^i v)$$

while for baryons it is enough to consider

$$m_0^2(\bar{u}u)+m_1^2(\bar{u}^3u_3)+m_2^2(\bar{u}\boldsymbol{\sigma}u)\,.\,(\bar{u}\boldsymbol{\sigma}u).$$

Taken between massless supermultiplet wavefunctions and comparing with physical masses we arrive at the estimates (in GeV units), $\mu_0^2 \simeq 0{\cdot}23$, $\mu_1^2 \simeq 0{\cdot}22$, $\mu_2^2 \simeq -0{\cdot}12$, $\mu_3^2 \simeq \mu_4^2 \simeq 0{\cdot}03$ and $m_0^2 \simeq 0{\cdot}42$, $m_1^2 \simeq 0{\cdot}43$, $m_2^2 \simeq 0{\cdot}23$.

This suggests that we collect terms in the form

$$M^2(\text{mesons}) \simeq 0{\cdot}23(\bar{u}u+\bar{v}v+\bar{u}^3u_3+\bar{v}^3v_3)-0{\cdot}12(\bar{u}\boldsymbol{\sigma}u)\,.\,(\bar{v}\boldsymbol{\sigma}v)+0{\cdot}06(\bar{u}\lambda^i v)(\bar{v}\lambda^i u)$$
$$M^2(\text{baryons}) \simeq 0{\cdot}42(\bar{u}u+\bar{u}^3u_3)+0{\cdot}23(\bar{u}\boldsymbol{\sigma}u)\,.\,(\bar{u}\boldsymbol{\sigma}u). \tag{2}$$

† Expanding the physical states in the exact formula $\langle \mathbf{0}|H(0)|\mathbf{0}\rangle = 2m^2$, as a series of eigenstates of the bare Hamiltonian H_0, leads naturally to a mass² formula if we insist on the same normalization for bosons and fermions. It is fatuous to pretend that the first-order formula gives anything but a rough hint of the bulk of the symmetry breaking. The work on SU(6) mass formulae has therefore only been used as a guide to H , and of necessity the mass relations can only be approximately valid to this order. If anything, the agreement in table 1 is probably too good.

‡ If we define creation operators $a^\dagger$ and $b^\dagger$ for quarks and antiquarks, so that a meson state is written

$$|p, a\lambda\overline{b\lambda'}\rangle = a^{a\lambda\dagger}(p)b^\dagger_{b\lambda'}(p)|0\rangle$$

then we can construct a meson field operator $\Phi_A^B(x)$ in the usual way,

$$\Phi_A^B(x) = \int \mathrm{d}p(\mathrm{e}^{-\mathrm{i}px}u_A^{a\lambda}(p)\bar{v}^B_{b\lambda}(p)a_{a\lambda}(p)b^{\dagger b\lambda'}(p)+\mathrm{hc})$$

such that its one particle matrix element is the momentum space wavefunction, $\langle 0|\Phi_A^B(0)|p\rangle = \Phi_A^B(p)$.

A similar construction, using a symmetrical product of three quarks, can be used to obtain baryon field operators. If now we take matrix elements of quark bilinears, or products thereof, the creation operators cancel out to leave us with products of wavefunctions. For instance, suppressing SU(3) labels,

$$\left\langle p_2,\lambda_2\overline{\lambda_2'}\left|\int \mathrm{d}x\bar{\psi}(x)\Gamma\psi(x)\right|p_1,\lambda_1\overline{\lambda_1'}\right\rangle$$
$$=\left\langle 0\left|a_{\lambda_2}(p_2)b^{\lambda_2'}(p_2)\left(\Sigma\int \mathrm{d}k a^{\dagger\lambda}(k)\bar{u}(k)\Gamma u(k)a_\lambda(k)+\ldots\right)a^{\dagger\lambda_1}(p_1)b^\dagger_{\lambda_1'}(p_1)\right|0\right\rangle$$
$$=\delta(p_1-p_2)(\bar{u}(p_1)\Gamma u(p_1)+\ldots).$$

It is in this sense that the mass tensor matrix elements written below are to be understood. The quark field $\psi(x)$ is indeed a second-quantized operator.

To show the reasonableness of the fit we exhibit in table 1 the experimental and calculated masses side by side.

Table 1. Physical against predicted hadron masses, using formula (1) or (2). Absence of $\underline{27}$ contributions allows singlet and octet masses to be derived by the Gell-Mann–Okubo relation. The D/F ratio which splits the Λ from the Σ cannot be predicted *a priori*

(Meson mass)^2		(Baryon mass)^2	
Experimental	Calculated	Experimental	Calculated
$\pi^2 = 0.02$	0.02	$N^2 = 0.88$	0.88
$K^2 = 0.24$	0.25	$\frac{1}{4}(3\Lambda^2+\Sigma^2) = 1.29$	1.30
$\eta_8^2 = 0.32$	0.33	$\Xi^2 = 1.73$	1.72
$\eta_0^2 = 0.90$	0.89	$\Delta^2 = 1.53$	1.49
$\rho^2 = 0.58$	0.58	$\Sigma^{*2} = 1.92$	1.91
$K^{*2} = 0.79$	0.81	$\Xi^{*2} = 2.34$	2.33
$\omega_8^2 = 0.87$	0.89	$\Omega^2 = 2.80$	2.75
$\omega_0^2 = 0.79$	0.73		

We can now try to rewrite these expressions in terms of quark fields. The first thing to remember is that $\bar{u}u$ and $\bar{v}v$ are SU(6) invariants (which could multiply any terms of the expansion without effect) and these are the large components of $\bar{\psi}\psi$ and $\bar{\psi}\gamma_0\psi$. By Lorentz invariance we are only entitled to consider functions of $\bar{\psi}\psi$ and of $(\bar{\psi}\gamma_\mu\psi)^2$, and we can discard the latter in so far as it would give rise to observable $(\bar{u}\boldsymbol{\sigma}v).(\bar{v}\boldsymbol{\sigma}u)$ corrections to M^2. For simplicity we will also suppose that the SU(6) singlet terms $(\bar{\psi}\psi)^2, (\bar{\psi}\psi)^3, \ldots$ are absent from H', an assumption which may need to be revised in the future because of the bearing of such interactions on chiral symmetry†. Recognizing that $\bar{u}v$ is the large component of $i\bar{\psi}\gamma_5\psi$ we can immediately identify $(\bar{u}\lambda^i v)(\bar{v}\lambda^i u)$ with $-\frac{1}{2}(\bar{\psi}\gamma_5\lambda^i\psi)^2$. The situation for $\bar{u}\boldsymbol{\sigma}u$ and $\bar{v}\boldsymbol{\sigma}v$ is much less definite. They are the large spatial components of $\bar{\psi}\sigma_{\mu\nu}\psi$ or $i\bar{\psi}\gamma_\mu\gamma_5\psi$, tensors which lead automatically to further components $\bar{u}\boldsymbol{\sigma}v$ or $\bar{u}v$ for their relativistic completions and which must be removed since they do not appear in M^2. The subtraction of these unwanted pieces is much easier to achieve for the axial rather than the tensor choice so we shall identify

$$(\bar{u}\boldsymbol{\sigma}u).(\bar{v}\boldsymbol{\sigma}v) \quad \text{with} \quad \tfrac{1}{2}\{(i\bar{\psi}\gamma_\mu\gamma_5\psi)^2+(\bar{\psi}\gamma_5\psi)^2\}$$

and

$$(\bar{u}\boldsymbol{\sigma}u).(\bar{u}\boldsymbol{\sigma}u) \quad \text{with} \quad -\{(i\bar{\psi}\gamma_\mu\gamma_5\psi)^2+(\bar{\psi}\gamma_5\psi)^2\}.$$

Apart from arguable aesthetic values, the choice we have made has the virtue of giving the same sign to the c' interaction term for mesons and baryons. We can thus summarize our conclusions for nonchiral SU(6)⊗SU(6) by formula (1). It is rather remarkable that two constants suffice to give a reasonable description of the masses. We think this result is valuable in its own right irrespective of the chiral arguments to follow.

In arriving at these SU(6) mass formulae we have taken linear representations of the nonchiral group corresponding to the finite nonunitary (unitary) basis of the relativistic U(6, 6) (U(12)) group provided by the spinor ψ. Now from ψ we can construct the usual vector and axial vector currents whose time components surely generate the chiral

† One can set up nonlinear realizations based entirely on quark fields. For instance, in a stereographic coordinate system, $\boldsymbol{\pi} = f(\bar{\psi}\gamma_5\boldsymbol{\tau}\psi)/(f^2-(\bar{\psi}\psi)^2)^{1/2}$ shows how $\bar{\psi}\psi$ can enter at a fundamental level.

SU(3)⊗SU(3) group because the interaction is supposed to be nonderivative. Furthermore this current algebra can be generalized to the full U(12) group if one adopts the hermitian currents

$$S^k = \tfrac{1}{2}\bar{\psi}\lambda^k\psi,\ T^k_{\mu\nu} = \tfrac{1}{2}\bar{\psi}\sigma_{\mu\nu}\lambda^k\psi,\ P^k = \tfrac{1}{2}\bar{\psi}\gamma_5\lambda^k\psi,\ V^k_\mu = \tfrac{1}{2}\bar{\psi}\gamma_\mu\lambda^k\psi,\ A^k = \tfrac{1}{2}\mathrm{i}\bar{\psi}\gamma_\mu\gamma_5\lambda^k\psi;\quad k = 0, 1, \ldots, 8 \tag{3}$$

and does a spatial integration to arrive at the appropriate hermitian generators. In terms of these bilinears,

$$H' = 4(\tfrac{2}{3})^{1/2}c(S^0 - 8^{-1/2}S^8) - 6c'(A^0_\mu A^0_\mu + P^0P^0 + \tfrac{1}{3}P^iP^i). \tag{4}$$

Now in chiral dynamics we may or may not require nonlinear realizations. A chiral transformation from the supermultiplet quark ψ to

$$q = \exp\{\mathrm{i}\lambda^k(\chi^k + \mathrm{i}\gamma_5\chi^k_5)\}\psi$$

will leave invariant the V and A currents,

$$V^k_\mu \pm A^k_\mu = \tfrac{1}{2}\bar{q}\gamma_\mu(1 \pm \mathrm{i}\gamma_5)\lambda^k q$$

but drastically alters to a nonlinear structure in χ the nature of hadrons viewed as q composites. Since we have taken the attitude that the ψ are effectively massless before introduction of the perturbation H', the possibility of having a chiral invariant mass term $m\bar{q}q$ in H_0 does not arise; it is therefore consistent, though by no means necessary, to suppose that the current quarks q and ψ are one and the same. With these reservations, we shall thus restrict ourselves to seeing how H' transforms linearly under SU(3)⊗SU(3). When we have a better idea of the connection (Gell-Mann 1972) between q and ψ it should be possible to improve on our work.

With respect to chiral SU(3) the Dirac bilinears transform as the multiplets,

$$S, T, P = (\bar{3}, 3)\oplus(3, \bar{3}), \qquad V, A = (8, 1)\oplus(1, 8)\oplus(1, 1).$$

Thus $A^0_\mu A^0_\mu$ is a (1, 1) chiral invariant, S will provide the traditional $(3, \bar{3})^{1+8}$ breaking and the P^2 terms give a *particular*

$$[(1, 1) + (3, \bar{3}) + (\bar{3}, 3) + (6, \bar{6}) + (\bar{6}, 6) + 8, 8)]^1$$

singlet contribution. We therefore contend that all symmetry breaking terms proposed so far are indeed present in H' but with well-defined relative coefficients provided by SU(6). Observe the ratio $-8^{-1/2}$ of scalar octet to scalar singlet both for mesons and baryons. The repercussions for chiral dynamics are almost self-evident and we shall only examine the most obvious ones here: mass formulae, scattering lengths and sigma terms, relying on the usual reduction methods for treating soft pseudoscalar mesons; in this connection it is worthwhile to point out that there need be no clash between chiral theory and supermultiplet theory, for in the limit as the meson mass μ vanishes, the supermultiplet wavefunction $\Phi(p) = (\mu + \gamma\,.\,p)(\gamma_5\phi_5 - \gamma_\lambda\phi_\lambda)$ shows the pseudoscalars to be axial derivatives (*PCAC*) and the vectors to be tensor derivatives (*PCTC*?).

The standard formula for pseudoscalar meson masses,

$$\tfrac{1}{2}f^2_k\mu^2_k\delta^{kl} = -\langle 0|[F^k_5, [F^l_5, H']]|0\rangle$$

using our H' and the commutators,

$$[F^k_5, S^l] = -\mathrm{i}d^{klm}P^m \qquad \text{and} \qquad [F^k_5, P^l] = \mathrm{i}d^{klm}S^m$$

gives the mass relations

$$\tfrac{1}{2}f_\pi^2\mu_\pi^2 = -0{\cdot}78\langle S^0\rangle + 0{\cdot}16\langle 6P^0P^0 - 2S^0S^0 + 7P^8P^8 - 11S^8S^8\rangle$$
$$\tfrac{1}{2}f_K^2\mu_K^2 = -1{\cdot}16\langle S^0\rangle + 0{\cdot}16\langle 6P^0P^0 - 2S^0S^0 + 7P^8P^8 - 11S^8S^8\rangle$$
$$\tfrac{1}{2}f_8^2\mu_8^2 = -1{\cdot}30\langle S^0\rangle + 0{\cdot}16\langle 6P^0P^0 - 2S^0S^0 + 7P^8P^8 - 11S^8S^8\rangle$$
$$\tfrac{1}{2}f_0^2\mu_0^2 = -1{\cdot}04\langle S^0\rangle + 0{\cdot}16\langle 6P^0P^0 - 6S^0S^0 + 16P^8P^8 - 16S^8S^8\rangle$$

if we assume the vacuum is SU(3) symmetric so that

$$\langle S^k\rangle = \delta^{k0}\langle S^0\rangle, \qquad \langle S^iS^j\rangle = \delta^{ij}\langle S^8S^8\rangle, \qquad \text{etc.}$$

The relevant vacuum expectation values can be evaluated if we set

$$f_\pi \simeq f_K \simeq f_8 \simeq f_0 = 0{\cdot}13\ \text{GeV}.$$

Thus $\langle S^0\rangle \simeq 0{\cdot}005$,

$$\langle 6P^0P^0 - 2S^0S^0 + 7P^8P^8 - 11S^8S^8\rangle \simeq -0{\cdot}025$$

and

$$\langle 6P^0P^0 - 6S^0S^0 + 16P^8P^8 - 16S^8S^8\rangle \simeq 0{\cdot}015.$$

At this stage we can say nothing about the individual values $\langle S^8S^8\rangle$ etc.

The pion scattering lengths a_0 and a_2 obtained via the relations

$$96\pi\mu_\pi a_0 = 5A + 32\mu_\pi^2/f_\pi, \qquad 48\pi\mu_\pi a_2 = A - 8\mu_\pi^2/f_\pi$$

and

$$-\tfrac{1}{4}f_\pi^4 A = \langle 0|[F_5^1,[F_5^1,[F_5^1,[F_5^1,H']]]]|0\rangle$$

provide more information, namely,

$$-\tfrac{1}{3}8\pi f_\pi^4\mu_\pi(a_0+2a_2) = 0{\cdot}78\langle S^0\rangle - 0{\cdot}11\langle 40P^0P^0 - 14S^0S^0 + 47P^8P^8 - 73S^8S^8\rangle.$$

Of the other four-point meson interactions, the Kπ scattering processes and the $\eta' \to \eta\pi\pi$ decay seem the most promising for giving further clues.

Turning to the baryons, the mass formulae are obtained from the expectation values $\langle\Psi|H'|\Psi\rangle$ where the bare states are massless and the unit baryon number sector of H' is considered. Thus,

$$m_\Psi^2 \simeq 1{\cdot}38\langle\Psi|S^0 - 8^{-1/2}S^8 - A_\mu^0A_\mu^0 - P^0P^0 - \tfrac{1}{3}P^iP^i|\Psi\rangle.$$

This is to be contrasted with the sigma term

$$\sigma_{\Psi\Psi}^{\pi\pi} = \tfrac{1}{3}\sum_{i=1}^{3}\langle\Psi|[F_5^i,[F_5^i,H']]|\Psi\rangle$$
$$\simeq 1{\cdot}38\langle\Psi|\tfrac{1}{2}S^0 + \tfrac{1}{4}\sqrt{(2)}S^8 + 8P^0P^0 - \tfrac{8}{3}S^0S^0 + 4P^1P^1 - \tfrac{28}{3}S^1S^1 + 4P^4P^4 - 4S^4S^4$$
$$+\tfrac{4}{3}P^8P^8 - \tfrac{4}{3}S^8S^8 + \tfrac{8}{3}\sqrt{2}\{P^0,P^8\} - \tfrac{4}{3}\sqrt{2}\{S^0,S^8\}|\Psi\rangle$$

where new expectation values like $\langle S^8S^8\rangle$ make an appearance. Thus at the level of linear chiral SU(3) the mass terms are independent of the sigma terms. Unless one can estimate these new scalar matrix elements, by saturating with one particle states say, no further progress can be made.

Another approach which suggests itself by our earlier considerations is to approximate massless vector mesons by the divergence of a tensor current, in contradistinction to vector dominance which relies on a field–current identity. Thus it may be possible to write $f_V^k \phi_\rho^k = \partial_\mu T_{\mu\rho}^k$ with T defined as in (3). Like the axial currents, we expect these tensor currents to be conserved when $H' = 0$, in the limit of zero supermultiplet mass (the strong binding limit where the $\mathrm{i}\gamma \cdot \partial$ term in the kinetic Lagrangian can be disregarded). Letting

$$F_r^k = \int T_{0r}^k \, \mathrm{d}^3 x$$

we obtain the time development by

$$[F_r^k, H'] = \mathrm{i}\dot{F}_r^k = \mathrm{i} \int \partial_\mu T_{\mu r}^k \, \mathrm{d}^3 x = \mathrm{i} f_V^k \int \phi_r^k \, \mathrm{d}^3 x$$

on a parallel with

$$[F_5^k, H'] = \mathrm{i}\dot{F}_5^k = \mathrm{i} \int \partial_\mu A_\mu^k \, \mathrm{d}^3 x = \mathrm{i} f_P^k \int \phi_5^k \, \mathrm{d}^3 x.$$

We would then obtain the vector meson masses via

$$\tfrac{1}{2} f_k^2 \mu_k^2 \delta^{kl} \delta_{rs} = -\langle 0 | [F_r^k, [F_s^l, H']] | 0 \rangle$$

by analogy with the pseudoscalar case. There are three difficulties to be overcome however: (i) there seems to be no way of measuring a fundamental tensor current in the same direct way that the axial enters in the weak interaction, (ii) extrapolations from physical to massless vector meson are likely to be unreliable, and (iii) new expectation values are generated by commutation with F_r which are not obviously related to the pseudoscalar expectation values.

In spite of these problems and ambiguities we believe that nonchiral SU(6) breaking will provide useful insight into the structure of chiral SU(3) breaking. Indeed once the relation between the current quarks and the supermultiplet quarks is clarified and quantified (Gell-Mann 1972) we are optimistic that the preliminary analysis we have presented above can be carried through to its logical end.

Acknowledgments

I have profited greatly from conversations with Drs P Dittner, S Eliezer, C J Isham, and H F Jones.

References

Akyeampong D 1966 *Nuovo Cim.* A **43** 499–506
Auvil P 1972 *Stanford preprint* SLAC-PUB-1078
Barnes K J and Isham C J 1970 *Nucl. Phys.* B **15** 333–49
Dittner P, Dondi P H and Eliezer S 1972 *Nucl. Phys.* B **49** 242
Fayyazzuddin and Rashid M A 1966 *Nuovo Cim.* A **42** 1000–3
Gell-Mann M 1972 *Conf. Summary Rep., 16th Int. Conf. on High Energy Physics, Chicago* to be published
Gell-Mann M, Oakes R J and Renner B 1968 *Phys. Rev.* **175** 2195–9
Genz H and Katz J 1970 *Nucl. Phys.* B **21** 333–45
Harari H and Rashid M A 1966 *Phys. Rev.* **143** 1354–8
Sirlin A and Weinstein M 1972 *NYU Technical Report* 4/72